

**INDIAN STATISTICAL INSTITUTE
CHENNAI CENTRE**

M. STAT.-I YEAR

ELECTIVE - DISCRETE MATHEMATICS

Time: 3 Hours

END SEMESTER EXAMINATION

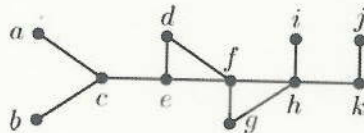
Marks : 50

Answer all the questions

1. Each user on computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there? (5 Marks)
2. During a month with 30 days, a football team plays at least one game a day, but no more than 45 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games. (6 Marks)
3. Words of length n , using only the three letters a, b, c , are to be transmitted over a communication channel subject to the condition that no word in which two a 's appear consecutively is to be transmitted. Determine the number of words allowed by the communication channel. (5 Marks)
4. Show that Stirling numbers of second kind satisfy the following: (6 Marks)
For $n \in \mathbb{Z}^+$, $S_{n,0} = 0$, $S_{n,n} = 1$, and $S_{n,k} = S_{n-1,k-1} + kS_{n-1,k}$, $0 < k < n$.
5. Using generating function, solve the recurrence relation: $a_n + a_{n-1} - 16a_{n-2} + 20a_{n-3} = 0$ subject to the initial values $a_0 = 0$, $a_1 = 1$, and $a_2 = -1$. (5 Marks)
6. Let D_n denote the number of derangements of $\{1, 2, \dots, n\}$. Show that (6 Marks)

$$D_n = n! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \frac{1}{n!} \right).$$

7. Use depth-first search to find a spanning tree of the graph G shown below, starting with the vertex f . (Draw all the intermediate trees). (5 Marks)



8. (a) State Max-flow-Min-cut theorem. (12 Marks)
(b) Using Ford-Fulkerson algorithm, find the maximum flow from s to t in the network given below. (Write all the intermediate steps).

